Algebraic Geometry WS20
Exercise set 1.

Instructor: Anton Mellit

Problem 1 (Hartshorne, p. 7, Ex. 1.1)**.** Let k be an algebraically closed field.

(1) Show that the ring $k[x,y]/(x^2 - y)$ is isomorphic to the ring $k[x]$.
(2) Show that the ring $k[x,y]/(xy - 1)$ is not isomorphic to the ring $k[x]$.
(3) Suppose $p \in k[x,y]$ is an irreducible polynomial of degree 2. Show that the ring $k[x,y]/p$ is isomorphic on of the two rings above.

Problem 2 (see Eisenbud, p. 49, Ex. 1.8)**.** Consider the maps Z and I between ideals in $k[x_1, \dots, x_n]$ and subsets of k^n. Show that Z and I provide bijections between the image of Z and the image of I.

Problem 3 (Eisenbud, p. 49, Ex. 1.10)**.** Describe rings whose real points correspond to the points of the following geometric shapes:

(1) The unit circle.
(2) Two lines crossing at a point.
(3) A parabola and a circle inside it so that the circle touches the tip of the parabola and there are no further intersection points.

Problem 4. Describe geometrically what points of the circle with coefficients in the ring $\mathbb{R}[x]/(x^2)$ are.

Due date: 16.10.2020, 9:45