Algebraic Geometry WS20
Exercise set 10.

Instructor: Anton Mellit

Problem 1. Let X be the blowup of $\mathbb{A}^2$ at the origin $(0,0)$ and let $\pi : X \to \mathbb{A}^2$ be the natural projection. Let $Y \subset \mathbb{A}^2$ be the union of the coordinate axes. Decompose the preimage $\pi^{-1}(Y)$ into irreducible components and compute their pairwise intersections.

Problem 2. Consider the subscheme $Y \subset \mathbb{A}^2$ given by the equation $x^3 + xy = y^2$. Construct a surjective morphism $\mathbb{A}^1 \to Y$. Is Y irreducible?

Problem 3. The same question as in Problem 1, but for Y from Problem 2.

Problem 4. Let $Q \subset \mathbb{P}^3$ be the quadric given by the homogeneous equation $xy = zw$. Using the idea of central projection from the last lecture construct a surjective morphism $f : \mathbb{P}^2 \to Q$. Show[1] that $\mathbb{P}^2$ is the blow-up of Q (this involves covering Q by affine charts U_i and describing the restriction of f to $f^{-1}(U_i)$).

Due date: 15.01.2021, 9:45

[1] It is a part of the problem to find out what this means.