Algebraic Geometry WS20
Exercise set 11.

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Problem 1. Let $Q \subset \mathbb{P}^3$ be the quadric given by the homogeneous equation $xy = zw$. Using the idea of central projection from the last lecture construct a surjective morphism $f : \mathbb{P}^2 \to Q$. Show[1] that $\mathbb{P}^2$ is the blow-up of Q (this involves covering Q by affine charts U_i and describing the restriction of f to $f^{-1}(U_i)$).

Problem 2. Let Z be the closed subscheme of $\mathbb{A}^4$ whose points over a ring R are 2×2 matrices with entries in R with determinant 0 and trace 1. Describe Z as the spectrum of a ring. Construct a surjective morphism $\pi : Z \to \mathbb{P}^1$. For any ring R describe morphisms $\mathrm{Spec}(R) \to \mathbb{P}^1$. Show that for any ring R any morphism $\varphi : \mathrm{Spec}(R) \to \mathbb{P}^1$ factors through Z, i.e. there exists $\psi : \mathrm{Spec}(R) \to Z$ such that $\pi \circ \psi = \varphi$.

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[1] It is a part of the problem to find out what this means.