Algebraic Geometry WS20
Exercise set 2.

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Some notations: k is a field, $A^n = \operatorname{Spec} k[x_1, \ldots, x_n]$.

Problem 1. Find all prime ideals of the ring of formal power series $k[[x]]$. The ring of power series is defined similarly to the ring of polynomials, but considering infinite sequences $(a_0, a_1, a_2, \ldots)$ which are written as $a_0 + a_1 x + a_2 x^2 + \cdots$.

Problem 2. Let k be a field, and let U, V be two non-empty open subsets of A^n.

(1) Show that the intersection $U \cap V$ is not empty.
(2) Show that $U \cap V$ contains a geometric point.

Problem 3 (Hartshorne, p. 7, Ex. 1.4)**.** Let k be an algebraically closed field. Consider the set k^2 with the following two topologies. The first one comes from identifying k^2 with $\operatorname{Specm} k[x, y]$, on which we have the Zariski topology. The second one is the product topology of $\operatorname{Specm} k[x] \times \operatorname{Specm} k[y]$. Show that the two topologies are different.

Problem 4. Let k be an algebraically closed field.

(1) Describe all prime ideals of the ring $k[x, y]/(xy)$.
(2) Describe all prime ideals of the ring $k[x, y, z]/(x^2 - yz, xz - x)$ (see Hartshorne, p. 7, Ex. 1.3).

Due date: 23.10.2020, 9:45