Algebraic Geometry WS20
Exercise set 3.

Instructor: Anton Mellit

Problem 1 (Hartshorne, p. 8, Ex. 1.6)**.** Let X be an irreducible topological space. Show that every open subset of X is irreducible. Show that if a subset $Y \subset X$ is irreducible (for the induced topology) then the closure $\bar{Y}$ is irreducible.

Problem 2 (Hartshorne, p. 8, Ex. 1.7)**.** A topological space is noetherian if any decreasing sequence of closed sets stops.

(1) Show that every open cover of a noetherian space has a finite subcover (this is called *quasi-compact*, while *compact* means Hausdorff and quasi-compact).
(2) Show that any subset of a noetherian space is noetherian for the induced topology.
(3) Show that a noetherian Hausdorff space must be a finite set with discrete topology.

Problem 3 (Hartshorne, p. 8, Ex. 1.10)**.**

(1) Let Y be a subset of a topological space X with the induced topology. Show that $\dim Y \leq \dim X$.
(2) In the above situation suppose $\dim Y = \dim X$. Show that $\bar{Y} = X$.
(3) If X is a topological space covered by open sets $\{U_\lambda\}_{\lambda\in\Lambda}$, show that $\dim X = \sup_{\lambda\in\Lambda} \dim U_\lambda$.

Problem 4 (Hartshorne, p. 8, Ex. 1.11)**.** Let k be algebraically closed. Consider the set $Y = \{(t^3, t^4, t^5) | t \in k\} \subset A^3$ and let $I = I(Y)$.

(1) Show that Y is irreducible.
(2) Find the dimension of Y.
(3) Show that I can be generated by 3 elements, but cannot be generated by 2 elements.
(4) Show that there exists ideal $J \subset k[x_1, x_2, x_3]$ generated by 2 elements such that I is among the set of minimal primes containing $\operatorname{rad} J = I$.

Due date: 30.10.2020, 9:45