Algebraic Geometry WS20
Exercise set 5.

Instructor: Anton Mellit

Problem 1. Let R be a ring and $\mathfrak{p} \subset R$ a prime ideal. Let $S \subset R$ be a multiplicative set. Show that the heights of $\mathfrak{p}$ and $S^{-1}\mathfrak{p}$ are equal.

Problem 2. Let R be a Noetherian domain. R is called a unique factorization domain (UFD) if every element $f \in R$ can be uniquely written as $f = uf_1 \dots f_n$ where u is invertible, $f_1, \dots, f_n$ are irreducible. The uniqueness is meant up to a permutation of $f_1, \dots, f_n$ and up to multiplying f_i by an invertible element. Show that R is a UFD if and only if every prime ideal of height 1 in R is principal.

Problem 3. Suppose R is a Noetherian domain. Show that $\dim R[x] = \dim R + 1$.

Due date: 13.11.2020, 9:45