Algebraic Geometry WS20
Exercise set 6.

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Problem 1. Describe all schemes which have exactly one point.

Problem 2. Describe all schemes which have exactly two points.

Problem 3 (Exercise I-25, Eisenbud-Harris "The Geometry Of Schemes", the smallest nonaffine scheme)**.** Let k be a field. Let X be the topological space with three points p, q_1, q_2. Topologize X by making $X_1 := \{p, q_1\}$ and $X_2 := \{p, q_2\}$ open sets. Set

$$O(X) = O(X_1) = O(X_2) = k[x]_{(x)}, \quad O(\{p\}) = k(x) \quad \text{(the rational functions)},$$

with restriction maps $O(X) \to O(X_i)$ the identity and $O(X_i) \to O(\{p\})$ the obvious inclusion for each $i = 1, 2$. Show that X is a ringed space and a scheme. Show that it is not an affine scheme.

Problem 4. Suppose X is an affine scheme and suppose $U \subset X$ is an open subset such that the corresponding open subscheme is affine. Is U necessarily a basic open subset?

Due date: 20.11.2020, 9:45