Algebraic Topology SS19
Exercise set 1.

Instructor: Anton Mellit

Problem 1. [Hatcher, Ch. 0, Ex. 1] Construct an explicit deformation retraction of the torus with one point removed onto a graph consisting of two circles intersecting in a point, namely longitude and meridian circles of the torus.

Problem 2. [Hatcher, Ch. 0, Ex. 2] Construct an explicit deformation retraction of $\mathbb{R}^n \setminus \{0\}$ onto S^{n-1}.

Problem 3. [Hatcher, Ch. 1.1, Ex. 5] Show that for a space X the following three conditions are equivalent:

(1) Every map $S^1 \to X$ is homotopic to a constant map.
(2) Every map $S^1 \to X$ extends to a map $D^2 \to X$.
(3) $\pi_1(X, x_0) = 0$ for all $x_0 \in X$.

Deduce that a path connected space X is simply-connected if and only if all maps $S^1 \to X$ are homotopic. (In this problem, 'homotopic' means 'homotopic without regard to basepoints').

Problem 4. [Hatcher, Theorem 1.9] Prove the Brouwer fixed point theorem in dimension 2, which says that any continuous map $h : D^2 \to D^2$ has a fixed point. Hint: assuming a map without fixed points exists, construct a map for D^2 to $\mathbb{R}^2 \setminus \{0\}$ which extend the identity map on the unit circle.

Problem* 5. [1] Suppose a map $f : S^1 \to S^1$ satisfies $f(-x) = -f(x)$ for all $x \in S^1$. Show that the class of f in $\pi_1(S^1)$ is not zero.

Due date: 19.03.2019

---

[1] Harder problems are marked with $*$