Algebraic Topology SS19
Exercise set 3.

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Problem 1. Let $\{G_\alpha\}_{\alpha\in\Lambda}$ be a collection of groups, and let F be the free product

$$F = *_{\alpha\in\Lambda} G_\alpha,$$

so elements of F are equivalence classes of sequences (g_1, α_1),...,(g_m, α_m). We call a sequence *reduced* if $g_i \neq 1$ and $\alpha_i \neq \alpha_{i+1}$ for all i. Construct a map from F to the set of reduced sequences and show that it is a bijection.

Problem 2. [Hatcher, p. 52, Ex. 1] Show that the free product $G * H$ of nontrivial groups G and H has trivial center, and that the only elements of $G * H$ of finite order are the conjugates of finite-order elements of G and H (Hint: use Problem 1).

Problem 3. [Hatcher, p. 52, Ex. 2] Let $X \subset \mathbb{R}^m$ be the union of convex open sets $X_1, \ldots, X_n$ satisfying $X_i \cap X_j \cap X_k \neq \varnothing$ for all i, j, k. Show that X is simply-connected.

Problem 4. [Hatcher, p. 53, Ex. 4] Let $X \subset \mathbb{R}^3$ be the union of n lines passing through the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$.

Problem 5. [Hatcher, p. 53, Ex. 7] Let X be the space obtained by identifying the north and the south poles of S^2 to a single point. Show that X is homeomorphic to a CW complex and compute its fundamental group.

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