Algebraic Topology SS19
Exercise set 4.

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Problem 1. [Hatcher, p. 20, Ex. 23] Suppose a CW complex X is a union of two contractible subcomplexes whose intersection is also contractible. Show that X is contractible. Is the statement true without the assumption on the intersection?

Problem 2. [Hatcher, Proposition 0.18] Suppose A is a subcomplex of a CW complex X and $f, g : A \to Y$ are homotopic. Show that $X \sqcup_f Y$ is homotopy equivalent to $X \sqcup_g Y$ relative to A. (Relative to A means that all the homotopies in the definition of homotopy equivalence are required to be identity on A.)

Problem 3. [Hatcher, p. 19, Ex. 19] Show that the space obtained from S^2 by attaching any n 2-cells along any collection of n circles in S^2 is homotopy equivalent to the wedge of $n + 1$ 2-spheres.

Problem 4. [Hatcher, p. 53, Ex. 8] Compute the fundamental group of the space obtained from two tori $S^1 \times S^1$ by identifying a circle $S^1 \times \{x_0\}$ in one torus with the corresponding circle $S^1 \times \{x_0\}$ in the other torus.

Problem 5. [Hatcher, p. 55, Ex. 21] Let X and Y be topological spaces. Their *join* $X*Y$ is defined by the quotient of $X \times Y \times I$ by the identifications $(x, y_1, 0) \sim (x, y_2, 0)$ and $(x_1, y, 1) \sim (x_2, y, 1)$ for all $x, x_1, x_2 \in X$ and $y, y_1, y_2 \in Y$. Equivalently, it is the pushout

$$\begin{array}{ccc} X \times Y \sqcup X \times Y & \xrightarrow{\mathrm{Id}_{X\times Y} \times \{0\} \sqcup \mathrm{Id}_{X\times Y} \times \{1\}} & X \times Y \times I \\ \downarrow{\scriptstyle \mathrm{pr}_1 \sqcup \mathrm{pr}_2} & & \\ X \sqcup Y. & & \end{array}$$

Suppose X and Y are connected. Show that $X * Y$ is simply connected.

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