Algebraic Topology SS19
Exercise set 5.

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Problem 1. [Hatcher, p. 53, Ex. 8] Compute the fundamental group of the space obtained from two tori $S^1 \times S^1$ by identifying a circle $S^1 \times \{x_0\}$ in one torus with the corresponding circle $S^1 \times \{x_0\}$ in the other torus.

Problem 2. [Hatcher, p. 79, Ex. 2] Show that if $p_1 : \tilde{X}_1 \to X_1$ and $p_2 : \tilde{X}_2 \to X_2$ are coverings then $p_1 \times p_2 : \tilde{X}_1 \times \tilde{X}_2 \to X_1 \times X_2$ is a covering too. Suppose X_1 and X_2 are path connected. Describe the corresponding action of the fundamental group of $\pi_1(X_1 \times X_2)$.

Problem 3. [Hatcher, p. 79, Ex. 8] Let X and Y be path connected, locally path connected, semilocally simply connected spaces. Suppose X and Y are homotopy equivalent. Prove that the corresponding universal covering spaces $\tilde{X}$ and $\tilde{Y}$ are homotopy equivalent.

Problem 4. [Hatcher, p. 79, Ex. 9] Show that if a path-connected, locally path-connected space X has finite fundamental group, then every map $X \to S^1$ is null-homotopic. (Hint: use the covering space $\mathbb{R} \to S^1$).

Problem 5. [Hatcher, p. 79, Ex. 5] Let X be the subspace of $\mathbb{R}$ consisting of the four sides of the square $[0, 1] \times [0, 1]$ together with the segments $\{1/m\} \times [0, 1]$ for $m = 2, 3, 4, \ldots$. Show that for any covering $\tilde{X} \to X$ there is a neighborhood of the left edge of X that lifts homeomorphically to $\tilde{X}$. Deduce that X has no universal covering space.

Due date: 30.04.2019