Algebraic Topology SS19
Exercise set 9.

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Problem 1. [Hatcher, Ex. 14, p. 132]Determine whether there exists a short exact sequence

$$0 \to \mathbb{Z}/4\mathbb{Z} \to \mathbb{Z}/8\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z} \to \mathbb{Z}/4\mathbb{Z} \to 0.$$

More generally, determine which abelian groups A fit into a short exact sequence

$$0 \to \mathbb{Z}/p^n\mathbb{Z} \to A \to \mathbb{Z}/p^m\mathbb{Z} \to 0$$

with p prime. What about the case of short exact sequences

$$0 \to \mathbb{Z} \to A \to \mathbb{Z}/n\mathbb{Z} \to 0?$$

Problem 2. [Hatcher, Ex. 17 (a), p. 132] Compute the reduced homology groups $\tilde{H}_n(X/A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.

Problem 3. [Hatcher, Ex. 20, p. 132] Show that $\tilde{H}_n(X) \approx \tilde{H}_{n+1}(SX)$ for all n, where SX is the suspension of X (recall that $SX = X \times I/(X \times \{0\})/(X \times \{1\})$).

Problem 4. [Hatcher, Ex. 22, p. 132] Prove by induction on dimension the following facts about the homology of a finite-dimensional CW complex X, using the observation that X^n/X^{n-1} is a wedge sum of n-spheres:

(1) If X has dimension n, then $H_i(X) = 0$ for $i > n$ and $H_n(X)$ is free.
(2) $H_n(X)$ is free with basis in bijective correspondence with the n-cells if there are no cells of dimensions $n-1$, $n+1$.
(3) If X has k n-cells, then $H_n(X)$ is generated by at most k elements.

Problem 5. [Hatcher, Ex. 26, p. 133] Compare $\tilde{H}_1(X/A)$ with $\tilde{H}_0(A)$ and conclude that $\tilde{H}_1(X/A)$ does not fit into an exact sequence

$$\tilde{H}_1(A) \to \tilde{H}_1(X) \to \tilde{H}_1(X/A) \to \tilde{H}_0(A) \to \tilde{H}_0(X)$$

where $X = [0,1]$ and A is the sequence $1, 1/2, 1/3, \ldots$ together with its limit 0. What happens if A is a finite subset of X instead?

Due date: 28.05.2019