Commutative algebra WS18
Exercise set 6.

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Problem 1. [AM, Proposition 2.1] Show that:

(1) If $N \subset M \subset L$ are R-modules, then $(L/N)/(M/N) \cong L/M$.
(2) If M_1, M_2 are submodules of M, then $(M_1 + M_2)/M_1 \cong M_2/(M_1 \cap M_2)$.

Problem 2. [AM, Ch. 2, Ex. 11] Let R be a ring $\neq 0$. Let $\phi : R^{\oplus m} \to R^{\oplus n}$ be a homomorphism of R-modules. Show that

(1) If ϕ is an isomorphism, then $m = n$.
(2) If ϕ is surjective, then $m \geq n$.

If ϕ is injective, is it always the case that $m \leq n$?

Problem 3. [AM, Ch. 2, Ex. 12] Let M be a finitely generated R-module and $\phi : M \to R^{\oplus n}$ a surjective homomorphism. Show that $\operatorname{Ker}\phi$ is finitely generated.

Problem 4. Let R be a Noetherian ring. Show that the ring of formal power series $R[[x]]$ is Noetherian.

Problem 5. [AM, Ch. 7, Ex. 11] Let R be a ring such that $R_{\mathfrak{p}}$ is Noetherian for every prime ideal $\mathfrak{p}$. Is R necessarily Noetherian?

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